Self-similar motion of laser half-space plasmas. II. Thermal wave and intermediate regimes

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The one-dimensional self-similar motion of an initially cold, half-space plasma of electron density n_0, produced by the (anomalous) absorption of a laser pulse of irradiation $\phi = \phi_0 t/\tau$ $(0 < t \leq \tau)$ at the critical density n_c, is considered; the analysis, which allows for electron heat conduction and ion-electron energy exchange, involves three dimensionless numbers: $\epsilon \equiv n_c/n_0$ assumed small, Z_i (ion charge number), and a parameter $\alpha \propto (n_0^2 \tau/\phi_0)^{2/3}$. For $\alpha \ll 1$, a weak discontinuity develops, separating a thermal wave (where convection is negligible) moving into the undisturbed plasma, from a much thinner isothermal flow expanding into the vacuum. For $1 \ll \alpha \ll \epsilon^{-4/3}$, there is an isentropic compression flow between a shock, bounding the undisturbed plasma, and a very thin transition layer bounding an expansion flow, much larger in extent than the compression region. In both regimes, the critical plane lies far in the expansion tail. The results break down when the density is so small that the plasma becomes collisionless. The analysis is also invalid for α too small. Using results previously found for $\alpha \gg \epsilon^{-4/3}$, a qualitative discussion of how plasma behavior changes with α, is given.

I. INTRODUCTION

In a previous paper[1] [hereafter called (I)] the authors analyzed the self-similar motion generated in a fully ionized, unmagnetized plasma (initially cold and occupying the half-space $x > 0$) by a laser pulse of irradiation $\phi \equiv \phi_0 t/\tau$ $(0 < t \leq \tau)$, uniformly absorbed in the plane where the electron density equals the critical density ($n_e = n_c \equiv m_e \omega^2/4\pi e^2$, ω being the laser frequency). The equations governing the problem were found to depend on three dimensionless numbers: the ion charge number Z_i; a parameter $\alpha \propto (n_0^2\tau/\phi_0)^{2/3}$, where n_0 is the undisturbed electron density;[2,3] and the ratio $n_c/n_0 \equiv \epsilon$ (assumed to be small). Detailed results were given in (I) for the regime $\alpha \gg \epsilon^{-4/3}$.

Here, we study the regimes $1 \ll \alpha \ll \epsilon^{-4/3}$ and $\alpha \ll 1$. We notice that defining $\alpha_c \equiv \alpha\epsilon^{4/3} \propto (n_c^2\tau/\phi_0)^{2/3}$, $(\alpha_c \ll \alpha)$, the three main regimes just mentioned correspond to conditions $1 \ll \alpha_c \ll \alpha$, $\alpha_c \ll 1 \ll \alpha$, and $\alpha_c \ll \alpha \ll 1$, respectively.

Section II briefly reviews the equations established in (I). Sections III and IV deal with the regimes $\alpha \ll 1$ and $1 \ll \alpha \ll \epsilon^{-4/3}$, respectively. In Sec. V we present a discussion of some results obtained and of the assumptions used in the analysis. Finally, Sec. VI carries out a qualitative discussion of how the plasma behavior changes when α sweeps from small to larger than $\epsilon^{-4/3}$ values through the intermediate regime $1 \ll \alpha \ll \epsilon^{-4/3}$ and the transition ranges $\alpha \sim 1$, $\alpha \sim \epsilon^{-4/3}$.

II. STATEMENT OF THE PROBLEM

Following (I), we assume quasi-neutrality, so that $n_e \simeq n_i Z_i \equiv n$, and $v_e \simeq v_i \equiv v$ (since the current density must vanish in the undisturbed plasma); however, we retain different temperatures for each species. We also assume (to start with) that the plasma is dominated by collisions, so that the electron heat conductivity and the ion-electron energy relaxation time may be written as[4]

$$K_e = \overline{K}_e T_e^{5/2}\ , \quad t_{ei} = \overline{t}_{ei} T_e^{3/2}/n_e\ ,$$

where $\overline{K}_e$ and $\overline{t}_{ei}$ are constant since we neglect the variations in all Coulomb logarithms; viscosities and ion conductivity are found numerically negligible.[3] As in (I), we introduce self-similar variables

$$\xi = x/[\tfrac{3}{4} v_0 \tau (t/\tau)^{4/3}]\ , \quad u(\xi) = v/[v_0 (t/\tau)^{1/3}]\ ,$$
$$\overline{n}(\xi) = n/n_0\ , \quad \theta_j = T_j/[T_0 (t/\tau)^{2/3}]\ \ (j = e, i)\ , \tag{1}$$

and arrive at the equations

$$\frac{d\overline{n}}{d\xi} = \frac{\overline{n}}{\xi - u}\frac{du}{d\xi}\ , \tag{2}$$

$$u - 4(\xi - u)\frac{du}{d\xi} = -\frac{\alpha}{\overline{n}}\frac{d}{d\xi}[\overline{n}(Z_i\theta_e + \theta_i)]\ , \tag{3}$$

$$\overline{n}\left[\theta_e\left(1 + \frac{4}{3}\frac{du}{d\xi}\right) - 2(\xi - u)\frac{d\theta_e}{d\xi}\right]$$
$$= \frac{d}{d\xi}\left(\chi\theta_e^{5/2}\frac{d\theta_e}{d\xi}\right) - 4.3b(Z_i)\alpha\overline{n}^2\frac{\theta_e - \theta_i}{\theta_e^{3/2}} + \delta(\xi - \xi_c)\ , \tag{4}$$

$$\frac{\overline{n}}{Z_i}\left[\theta_i\left(1 + \frac{4}{3}\frac{du}{d\xi}\right) - 2(\xi - u)\frac{d\theta_i}{d\xi}\right] = 4.3b(Z_i)\alpha\overline{n}^2\frac{\theta_e - \theta_i}{\theta_e^{3/2}}\ , \tag{5}$$

$$\overline{n}(\xi_c) = n_c/n_0 \equiv \epsilon\ . \tag{6}$$

Electron inertia has been neglected in the momentum equation for the ion-electron fluid, from which the electric field has disappeared because of quasi-neutrality.[4] Expressions for v_0, T_0, α, and b are given in (I).

The boundary conditions are: in the undisturbed plasma ($\xi = \infty$):

$$u = \theta_e = \theta_i = 0\ , \quad \overline{n} = 1\ ; \tag{7}$$

at the vacuum-plasma boundary ($\xi = \xi_v$):

$$\overline{n} = 0\ , \quad u = \xi_v\ . \tag{8}$$

In addition, since the mean-free-path is proportional to $\theta_e^2/\overline{n}$, θ_e should vanish at ξ_v for a collision-dominated plasma. We advance, however, that, for the regimes here considered, the plasma will be found to be collisionless near ξ_v. Thus, we first simply require that the heat flux vanish at ξ_v, and second, include a heat flux limiter χ in Eq. (4)[5,6]

$$\chi = \left[1 + \left(\frac{8\pi m_e}{9k^3}\right)^{1/2} \frac{\bar{K}_e}{n_e} T_e \left|\frac{dT_e}{dx}\right|\right]^{-1} = \left(1 + \frac{0.059}{\alpha^{1/2} A_i^{1/2}} \frac{\theta_e}{\bar{n}} \left|\frac{d\theta_e}{d\xi}\right|\right)^{-1}, \tag{9}$$

where k is Boltzmann's constant and A_i is the ion mass number.

III. ASYMPTOTIC SOLUTION FOR $\alpha \ll 1$

A. Thermal wave region

In Ref. 3, the motion resulting from deposition of energy at the plane $\xi = 0$, within an *unbounded*, initially cold and uniform, plasma, was considered; for α small, a thermal wave was found to move into the undisturbed plasma, convection and ion temperature being negligible. Following the analysis there we expand the dependent variables in Eqs. (2)–(5) in powers of α and find to lowest order

$$u \sim \bar{n} - 1 \sim \theta_i \sim O(\alpha), \quad \theta_e \sim O(1),$$

the equation for θ_e reading

$$\theta_e - 2\xi\, d\theta_e/d\xi = d(\theta_e^{5/2}\, d\theta_e/d\xi)/d\xi, \tag{10}$$

where we assumed $\chi \simeq 1$.

Since the front ξ_f of the thermal wave must lie at a finite distance,[7] boundary conditions at $\xi = \infty$ are useless; however, vanishing of both electron heat flux and temperature at ξ_f are now required. In addition, we have

$$\theta_e^{5/2}\, d\theta_e/d\xi = -1 \quad \text{at } \xi = 0^+ \tag{11}$$

since, as we shall see, the thickness of the expanding plasma in the negative-ξ half-space is $O(\alpha^{1/2})$ and therefore, most of the energy is carried in the thermal wave; then, first, in a scale where $\xi = 0(1)$ the critical plane lies at the origin, and second, the leftward heat flux at that plane is negligible. The appropriate solution to Eq. (10) is given in Fig. 1, and differs from that in Ref. 3 because of condition (11).

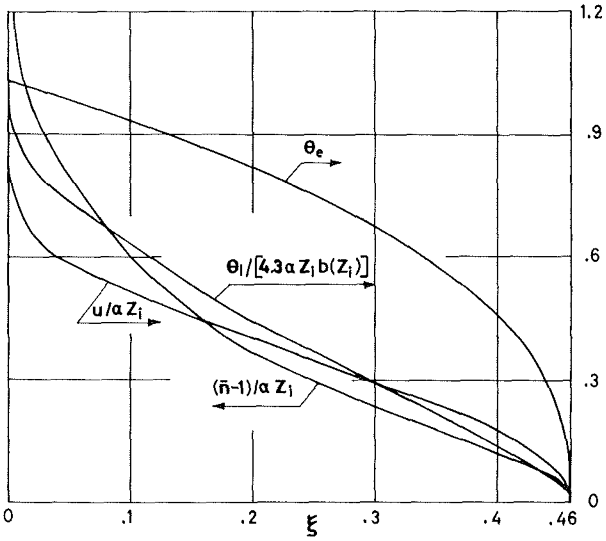

FIG. 1. Dimensionless temperatures, density, and velocity vs dimensionless distance in the thermal wave for $\alpha \ll 1$.

To dominant order, Eqs. (2), (3), and (5) read

$$d\bar{n}/d\xi = \xi^{-1}\, du/d\xi, \tag{12}$$

$$u - 4\xi\, du/d\xi = -\alpha Z_i\, d\theta_e/d\xi, \tag{13}$$

$$\theta_i - 2\xi\, d\theta_i/d\xi = 4.3\alpha Z_i b \theta_e^{-1/2}, \tag{14}$$

with boundary conditions

$$u = \bar{n} - 1 = \theta_i = 0 \quad \text{at } \xi = \xi_f. \tag{15}$$

Solutions to Eqs. (12)–(15) are given in Fig. 1 for $\xi > 0$.

We notice that at $\xi = 0$ both u and θ_i present a weak discontinuity (cusp) while $\bar{n} - 1$ goes to infinity. Actually, when $\xi = O(\alpha^{1/2})$ the pressure gradient due to density variations, neglected in (13), must be retained. The analysis of this thin layer would be then similar to that carried out in Ref. 3: The cusp is found to move to $\xi = (1/2)[\theta_e(0) Z_i \alpha]^{1/2}$. To its right u is determined by matching to the solution in Fig. 1; its value at the cusp is $u(\xi = 0)$ as given in that figure, while $\bar{n} - 1$ reaches a finite value of order $\alpha^{5/8}$. On the other hand, the behavior to the left of the cusp depends on the boundary conditions at ξ_v, and bears no resemblance to the results in Ref. 3.

B. Isothermal expansion region

Boundary conditions (8) imply that u and ξ are of the same order, while $\bar{n} = 0(1)$. It follows clearly from Eq. (4) that

$$\theta_e \simeq \text{const} = \theta_e(0).$$

Now defining

$$\hat{u} = u/[\alpha\theta_e(0) Z_i]^{1/2}, \tag{16}$$

$$\hat{\xi} = \xi/[\alpha\theta_e(0) Z_i]^{1/2}, \tag{17}$$

$$\hat{\theta}_i = \theta_i/\alpha b Z_i, \tag{18}$$

Eqs. (2), (3), and (5) become

$$\frac{d\bar{n}}{d\hat{\xi}} = \frac{\bar{n}}{\hat{\xi} - \hat{u}} \frac{d\hat{u}}{d\hat{\xi}}, \tag{19}$$

$$\hat{u} - 4(\hat{\xi} - \hat{u}) \frac{d\hat{u}}{d\hat{\xi}} = \frac{-1}{\bar{n}} \frac{d\bar{n}}{d\hat{\xi}}, \tag{20}$$

$$\hat{\theta}_i \left(1 + \frac{4}{3} \frac{d\hat{u}}{d\hat{\xi}}\right) - 2(\hat{\xi} - \hat{u}) \frac{d\hat{\theta}_i}{d\hat{\xi}} = \frac{4.3\bar{n}}{[\theta_e(0)]^{1/2}}. \tag{21}$$

The boundary conditions are

$$\bar{n} = 0, \quad \hat{u} = \hat{\xi}_v \quad \text{at } \hat{\xi} = \hat{\xi}_v \tag{22}$$

$$\bar{n} = 1, \quad \hat{u} = 0, \quad \hat{\theta}_i = \frac{\theta_i}{\alpha b Z_i} \simeq 4.27 \text{ at } \hat{\xi} = \tfrac{1}{2}, \tag{23}$$

the value of θ_i at the cusp clearly being $\theta_i(\xi = 0)$ as given in Fig. 1; on the other hand, the conditions on $\hat{u}$ and $\bar{n}$ at $\hat{\xi} = 1/2$ follow from the fact that at the cusp $\hat{u} = O(\alpha^{1/2})$ and $\bar{n} - 1 = O(\alpha^{5/8})$.

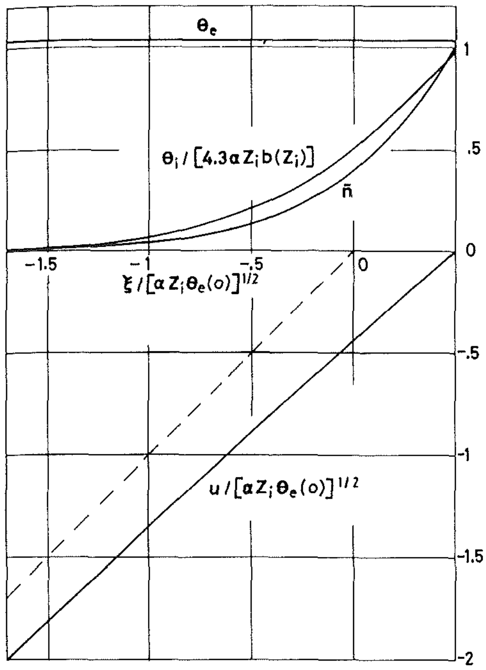

FIG. 2. Dimensionless temperatures, density, and velocity vs dimensionless distance in the isothermal expansion for $\alpha \ll 1$.

We now notice that once $\hat{u}$ and $\bar{n}$ have been obtained $\hat{\theta}_i$ follows by direct integration. Moreover, a single equation for $\hat{u}$ may be immediately obtained from (19) and (20):

$$\frac{d\hat{u}}{d\hat{\xi}} = \frac{-\hat{u}(\hat{\xi}-\hat{u})}{1-4(\hat{\xi}-\hat{u})^2}, \tag{24}$$

$$\hat{u}=0 \text{ at } \hat{\xi}=1/2 . \tag{25}$$

It should be pointed out, however, that $\hat{\xi}=1/2$, $\hat{u}=0$ is a nodal point of (24), and therefore, there are an infinite number of solutions to Eqs. (24)–(25). Now, for a particular solution, Eq. (19) may be integrated, starting at $\hat{\xi}=1/2$, $\bar{n}=1$: clearly, the solution looked for is the one for which $\bar{n}=0$ when $\hat{u}=\hat{\xi}$. Finally we find that this solution meets the line $\hat{u}=\hat{\xi}$ at $\hat{\xi}=-\infty$; all other solutions to Eqs. (24)–(25) either cross that line at a finite $\hat{\xi}$, where $\bar{n}\neq 0$, or do not meet it at all.

Figure 2 shows all three $\hat{u}$, $\bar{n}$, and $\hat{\theta}_i$ throughout the expansion layer; although, asymptotically, $\hat{\xi}_c \to -\infty$ as $\epsilon \to 0$, $\bar{n}(\hat{\xi})$ decreases so fast that, for any reasonable ϵ, $\hat{\xi}_c$ has a moderate value. Actually, as we shall see in Sec. V, the solution shown in Fig. 2 breaks down when $\bar{n}$ becomes small because the plasma becomes collisionless.

IV. ASYMPTOTIC SOLUTION FOR $1 \ll \alpha \ll \epsilon^{-4/3}$

A. Isentropic compression region

It was shown in (I) that as long as α is large there exists a region of isentropic compression behind a shock bounding the undisturbed plasma; its analysis here would be similar to that carried out in (I). In fact, if $\bar{n}_f=4$, $u_f=3\xi_f/4$, and $\theta_f=3\xi_f^2/4\alpha(Z_i+1)$ are the values behind the shock, as obtained from the jump conditions (ξ_f being the unknown shock position), the solution given in Fig. 2 of (I) in normalized variables

$$\eta \equiv \xi/\xi_f , \quad \nu \equiv \bar{n}/\bar{n}_f ,$$
$$y \equiv u/u_f , \quad z_j \equiv \theta_j/\theta_f \quad (z_i \simeq z_e \equiv z) , \tag{26}$$

remains valid for the present regime. It was found in (I) that the isentropic solution ceases to be valid near a point $\bar{\eta}(\bar{\eta}\simeq 0.82)$, where

$$y \simeq \tfrac{4}{3}\bar{\eta} - \tfrac{2}{5}(\eta-\bar{\eta}) , \quad z \simeq B_1(\eta-\bar{\eta})^{3/13} , \quad \nu \simeq B_2(\eta-\bar{\eta})^{-3/13}$$

($B_1 \simeq 1.70$, $B_2 \simeq 0.78$), the plasma becoming highly dense and cold.

B. Expansion region

We shall now make the ansatz that (a) to analyze the entire region between η_v and $\bar{\eta}$ it suffices to scale the variables once, and that (b) $\nu \leq O(1)$. We then notice first, that

$$\nu z = O(1) \tag{27}$$

since it must have the value B_1B_2 at $\bar{\eta}$, and second that condition (8) implies

$$y = O(\eta) . \tag{28}$$

Next, integrating Eq. (4) across a thin layer centered at the critical plane, and using the variables defined in (26), we get

$$\eta = O(\alpha^{-7/2}\xi_f^6 z^{7/2}) . \tag{29}$$

In addition, the condition of vanishing momentum for the entire plasma leads to

$$\nu y \eta = O(1) \tag{30}$$

since in the compression region $\int_{\bar{\eta}}^1 \nu y \, d\eta = O(1)$. Finally, the integral energy conservation law, which follows from the system (2)–(5), may be written as

$$\int_{\xi_v}^{\xi_f} 3\bar{n}(Z_i\theta_e + \theta_i + 4u^2/3\alpha)\, d\xi = Z_i ;$$

this equation, using (26) and $\nu \leq 0(1)$, shows that

$$\xi_f^3 \eta = O(\alpha) . \tag{31}$$

Equations (27)–(31) lead to $\xi_f = O(\alpha^{5/24})$ and suggest defining new variables

$$\hat{\eta} = \xi_f^{3/5}\alpha^{-1/2}\eta , \quad \hat{y} = \xi_f^{3/5}\alpha^{-1/2}y$$
$$\hat{\nu} = \xi_f^{-6/5}\alpha\nu , \quad \hat{z}_j = \xi_f^{6/5}\alpha^{-1}z_j . \tag{32}$$

Then, Eqs. (2)–(5) become

$$\frac{d\hat{\nu}}{d\hat{\eta}} = \frac{3\hat{\nu}}{4\hat{\eta}-3\hat{y}} \frac{d\hat{y}}{d\hat{\eta}} , \tag{33}$$

$$\hat{\nu}\hat{y} - \hat{\nu}(4\hat{\eta}-3\hat{y}) \frac{d\hat{y}}{d\hat{\eta}} = -\frac{d}{d\hat{\eta}}\left(\hat{\nu}\, \frac{Z_i\hat{z}_e+\hat{z}_e}{Z_i+1}\right) , \tag{34}$$

$$\frac{\hat{\nu}}{4}\left[\hat{z}_e\left(1+\frac{d\hat{y}}{d\hat{\eta}}\right) - \frac{4\hat{\eta}-3\hat{y}}{2}\frac{d\hat{z}_e}{d\hat{\eta}}\right]$$
$$= \frac{1}{16}\left(\frac{3}{4(Z_i+1)}\right)^{5/2} \frac{d}{d\hat{\eta}}\, \hat{z}_e^{5/2}\, \frac{d\hat{z}_e}{d\hat{\eta}}$$

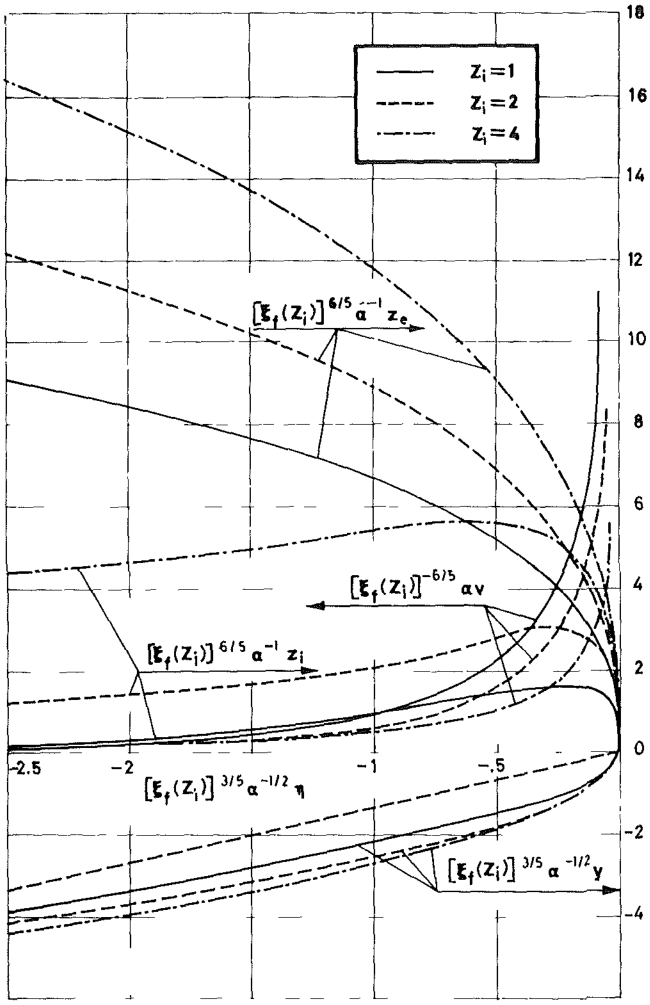

FIG. 3. Temperatures, density, and velocity normalized with the values behind the shock in the expansion region for $1 \ll \alpha \ll \epsilon^{-4/3}$.

$$-4.3b\left(\frac{4(Z_i+1)}{3}\right)^{3/2}\hat{\nu}^2\,\frac{\hat{z}_e-\hat{z}_i}{\hat{z}_e^{3/2}}, \tag{35}$$

$$\frac{\hat{z}_i}{4}\left(1+\frac{d\hat{y}}{d\hat{\eta}}\right)-\frac{4\hat{\eta}-3\hat{y}}{8}\,\frac{d\hat{z}_i}{d\hat{\eta}}=4.3b\left[\frac{4(Z_i+1)}{3}\right]^{3/2} Z_i\,\hat{\nu}\,\frac{\hat{z}_e-\hat{z}_i}{\hat{z}_e^{3/2}}; \tag{36}$$

explicit use of ξ_f in (32) has allowed us to drop it from Eqs. (33)–(36).

This system must be solved subject to five boundary conditions: these are

$$\hat{y}=\hat{z}_e=\hat{z}_i=0\ ,\quad \hat{\nu}\hat{z}_e=B_1B_2 \ \text{at}\ \hat{\eta}=\xi_f^{3/5}\alpha^{-1/2}\bar{\eta}\simeq 0, \tag{37}$$

(which follow from matching to the isentropic compression solution), together with the vacuum-plasma boundary condition,

$$\bar{n}=0 \ \text{when}\ 4\hat{\eta}=3\hat{y}. \tag{38}$$

There is no δ-function term in (35) because $\hat{\nu}_c=O(\alpha^{3/4}\epsilon)$, that is, $\hat{\eta}_c\to-\infty$ as $\alpha\epsilon^{4/3}\to 0$: the critical plane lies at $-\infty$ (we find $\hat{\eta}_v=-\infty$ as in the $\alpha\ll 1$ regime). However, once the system (33)–(38) has been solved, equating the resulting heat flux at $\hat{\eta}=-\infty$, to the energy flux deposited at the critical plane,

$$-\frac{1}{16}\left(\frac{3}{4(Z_i+1)}\right)^{5/2}\hat{z}_e^{5/2}\,\frac{d\hat{z}_e}{d\hat{\eta}}\bigg|_{\hat{\eta}\to-\infty}=\frac{Z_i+1}{12}\,\frac{\alpha^{1/2}}{\xi_f^{12/5}}$$

allows us to determine ξ_f, and thus to complete the solution; we notice that the system (33)–(38) indeed leads to $\hat{z}_e\sim(-\hat{\eta})^{2/7}$ as $\hat{\eta}\to-\infty$.

It may be shown analytically that in the neighborhood of $\hat{\eta}=0$, any solution to Eqs. (33)–(37) behaves as

$$\hat{z}_e\simeq A(-\hat{\eta})^{2/5}\ ,\quad \hat{y}\simeq-\frac{A^{7/2}}{25B_1B_2}\left(\frac{3}{4(Z_i+1)}\right)^{5/2}\frac{Z_i(-\hat{\eta})^{2/5}}{Z_i+1}\ ,$$

$$\hat{\nu}\simeq\frac{B_1B_2}{A}(-\hat{\eta})^{-2/5}\ ,$$

$$\hat{z}_i\simeq\hat{z}_e-\left(\frac{3}{4(Z_i+1)}\right)^4\frac{A^7(-\hat{\eta})^{4/5}}{430b(B_1B_2)^2}\,\frac{1}{Z_i+1}\ ,$$

where A is an arbitrary constant; it is possible to establish some bounds on A. Then, sweeping through the A range one arrives at the value that allows satisfying condition (38). The resulting solution for the expansion flow is shown in Fig. 3 for $Z_i=1$, 2, and 4. We find that

$$\alpha^{-5/24}\xi_f=0.39,\ 0.43,\ \text{and}\ 0.49\ ,$$

for $Z_i=1$, 2, and 4, respectively.

We finally notice that, as in (I), there exists a very thin transition layer centered at $\bar{\eta}$, where the density peaks; its analysis would be similar to that carried out in the Appendix of (I). We find $z=O(\alpha^{9/80})$, $\nu=O(\alpha^{-9/80})$ in this layer.

V. VALIDITY OF THE RESULTS

In the analysis, we assumed near-Maxwellian distribution functions, that is, $\lambda_j\ll\Delta x$ and $t_j\ll t$, where λ_j and t_j are the mean-free-path and collision time for species j, Δx is the characteristic width of a particular region of the motion, and $t\lesssim\tau$. In self-similar variables the first condition is

$$\theta_j^2/\bar{n}\Delta\xi\ll(\alpha m_i/m_e)^{1/2}\ , \tag{39}$$

and is, naturally, equivalent to the requirement that no flux limiter be needed [see Eq. (9)]; the second condition, on the other hand, is

$$\theta_e^{3/2}/\bar{n}\ll\alpha m_i/m_e\ , \tag{40}$$

$$\theta_i^{3/2}/\bar{n}\ll\alpha(m_i/m_e)^{1/2}\ , \tag{41}$$

for electrons and ions, respectively.

For $1\ll\alpha\ll\epsilon^{-4/3}$, the results of Sec. IV show that all three conditions (39)–(41) are well satisfied [implying in the more restrictive case, that $(m_i/m_e)^{1/2}$ be large]. For $\alpha\ll 1$, the most stringent requirement is found to be (see Sec. III) inequality (39) for electrons, in both the thermal wave and expansion regions; for the thermal wave, (39) leads to

$$1\ll(\alpha m_i/m_e)^{1/2}\ ,$$

while for the expansion we get

$$1\ll\alpha(m_i/m_e)^{1/2}\ ,$$

an even stronger condition. Thus, the analysis of Sec. III for the expansion has a very limited range of validity,

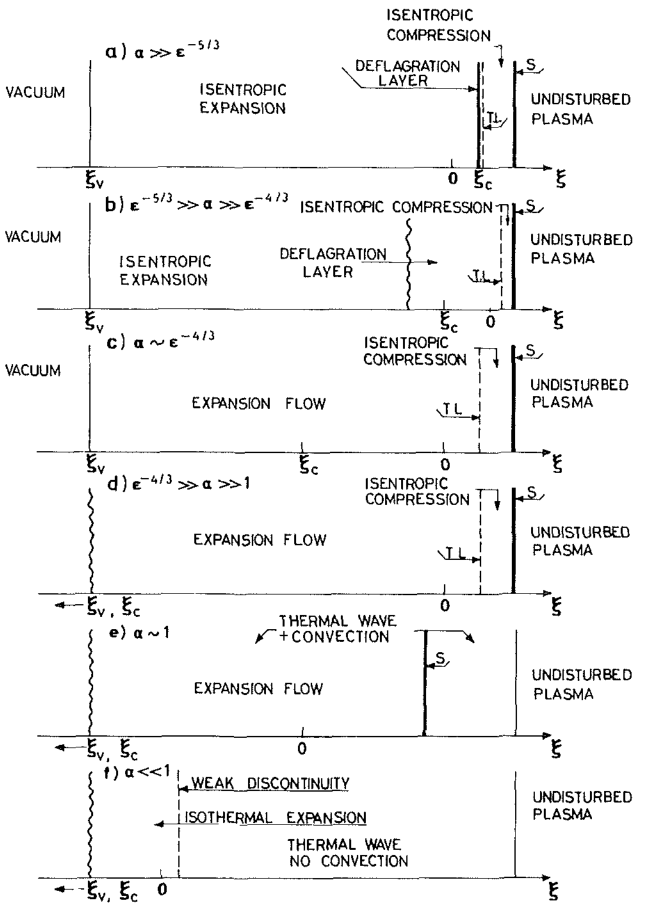

FIG. 4. Schematic representation of plasma behavior for different α regimes.

$(m_e/m_i)^{1/2} \ll \alpha \ll 1$. Clearly, for $\alpha \lesssim (m_e/m_i)^{1/2}$, the heat flux limiter given in Eq. (9) should be used. Actually, the flux limiter is even needed for the regimes $\alpha \ll 1$ and $1 \ll \alpha \ll \epsilon^{-4/3}$, in the tail of the expansion where $\bar{n}$ is small enough.

The hypothesis of quasi-neutrality, $\Delta x \ll \lambda_D$ (Debye length), used throughout the analysis, when written in self-similar variables reads

$$\omega\tau(t/\tau) \gg (A_i \epsilon \alpha \theta_e/\bar{n})^{1/2}(10/\Delta\xi) \; ; \qquad (42)$$

it is easily verified that for all reasonable values of ω, τ, and A_i, (42) is well satisfied except at the very beginning of the pulse (or far in the expansion tail, where $\bar{n}$ is extremely small).

VI. GENERAL DISCUSSION FOR ARBITRARY α

We are now in a position, using the results obtained here and in (I), to discuss how the plasma behavior changes as α goes from large to small values. Figures 4(a) and (b) correspond to the regime $\alpha \gg \epsilon^{-4/3}$ analyzed in (I). In Fig. 4(a) we consider the case $\alpha \gg \epsilon^{-5/3}$; there is then, beginning from the right, undisturbed plasma, a shock, a region of isentropic compression, a thin deflagration layer and finally, a much wider isentropic expansion, bounded by the vacuum at a finite distance from the origin ξ_v. The critical plane lies in the deflagration layer, to the right of the origin ($\xi_c > 0$). In Fig. 4(b) we consider the case $\epsilon^{-4/3} \ll \alpha \ll \epsilon^{-5/3}$, the only changes with respect to Fig. 4(a), being that now the deflagration is much wider than the compression region and ξ_c is negative. Clearly, if $\alpha \sim \epsilon^{-5/3}$, the compression flow and the deflagration layer have the same size.

Figure 4(c) for $\alpha \sim \epsilon^{-4/3}$ may be inferred as the intermediate limit from the results obtained in (I) and in Sec. IV. The main change is the merging of the deflagration and expansion regions (which thus ceases to be isentropic); the critical plane lies at a finite distance within it. Figure 4(d) corresponds to the regime $\epsilon^{-4/3} \gg \alpha \gg 1$ and is a schematic representation of the results of Sec. IV, the only change being that the plasma vacuum boundary lies at infinity (the density within the expansion region is much larger than the critical density). We notice that throughout Figs. 4(a)–(d) there is a transition layer (where the temperature is very low) just to the left of the isentropic compression, and an electron precursor ahead of the shock,[7] both being very thin.

Figure 4(e) for $\alpha \sim 1$ may be inferred as an intermediate limit from the results of Secs. III and IV. As α decreases to values of order unity, the precursor thickness and the temperature minimum grow until the temperature is of the same order everywhere and the shock stands in the middle of a quasi-thermal wave (convection being important, although there is isentropic flow nowhere). As α goes on decreasing, the shock moves to the origin and its intensity weakens. Figure 4(f) finally shows the regime $\alpha \ll 1$; the shock has become a weak discontinuity close to the origin, convection being negligible ahead of it. The expansion flow has become isothermal (electron conduction being dominant) and thin compared with the thermal wave. Throughout Figs. 4(d)–(f) the critical plane lies far in the expansion tail which reaches infinity. Actually, as noticed in Sec. V, the plasma is collisionless in the far tail, and therefore, the validity of the results breaks down there.

ACKNOWLEDGMENT

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